Generalized Score Distribution

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Model for Single PVS

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So we can validate what is the relation between ψ_o and ψ_u.

Theory

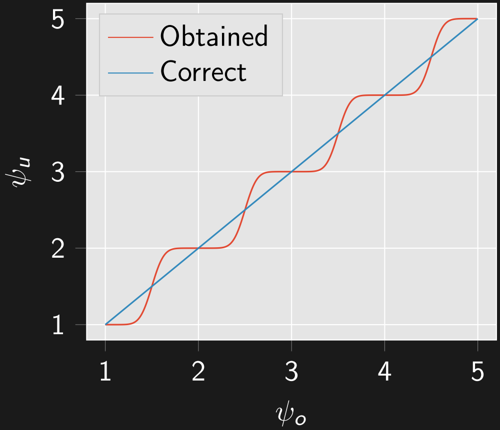
$$O_{ij} \sim \mathcal{N}(\psi_o, \sigma_o)$$

$$P(U_{ij} = k) = \begin{cases} \int_{-\infty}^{1.5} \frac{1}{\sqrt{2\pi}\sigma_o} e^{-\frac{(o-\psi_o)^2}{2\sigma_o}} & k = 1 \\ \int_{k-0.5}^{k+0.5} \frac{1}{\sqrt{2\pi}\sigma_o} e^{-\frac{(o-\psi_o)^2}{2\sigma_o}} & k \in \{2, 3, 4\} \\ \int_{4.5}^{\infty} \frac{1}{\sqrt{2\pi}\sigma_o} e^{-\frac{(o-\psi_o)^2}{2\sigma_o}} & k = 5 \end{cases}$$

Knowing $P(U_{ij} = k)$ we can calculate ψ_u and plot function $\psi_u(\psi_o)$ it should be $y = x$.

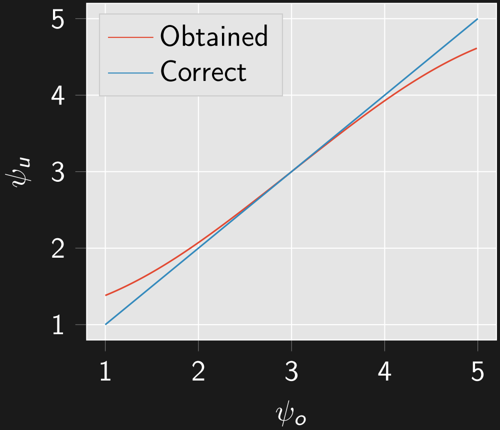
Small Standard Deviation

Let as assume that $\sigma_o = 0.1$.

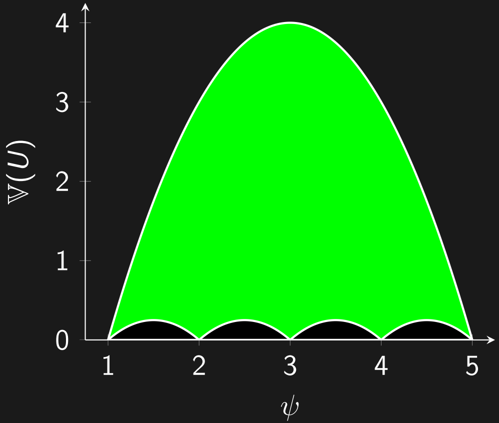


Large Standard Deviation

Let as assume that $\sigma_o = 1$.



Variance Limitation



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$$U \sim GSD(\psi, \rho)$$

- ψ specify the mean
- ρ specify the answers spread
  - $\rho = 1$ is the minimum variance so only $\lceil \psi \rceil$ and $\lfloor \psi \rfloor$ are possible
  - $\rho = 0$ is the maximum variance so only the minimum and the maximum value is possible

Existing Distributions

$\psi = 3.0$

0

4

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Legend:

- ○ $V_{\min}(\psi)$ - Bernoulli distribution,

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0 1 4

Legend:

- $\circ$ $V_{\min}(\psi)$ - Bernoulli distribution,
- $\times$ $V_{\text{Bin}}(\psi)$ - Binomial distribution
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- green line: BetaBinomial distribution
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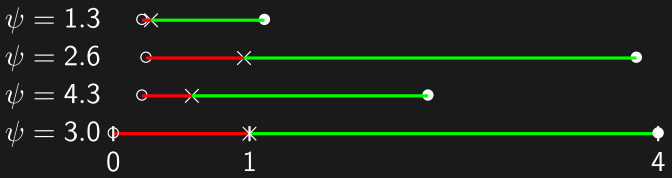

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$$P(X = 1) = pBer(X = 1) + (1 - p)Bin(X = 1)$$

$$P(X = 1) = p(2 - \psi) + (1 - p)\binom{4}{0}\left(\frac{\psi - 1}{4}\right)^4$$

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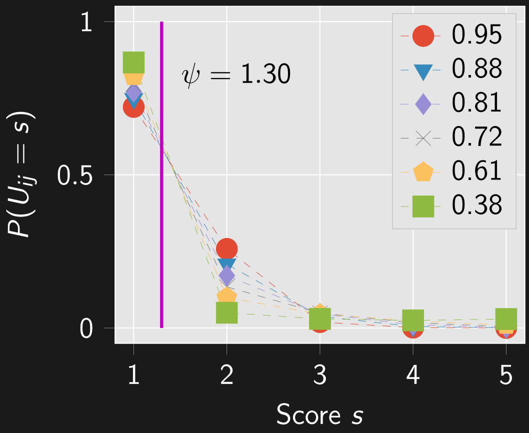
$$P_{G_\rho}(\epsilon = k - \psi) = \binom{4}{k-1} \times \frac{\mathcal{B}\left(\frac{\rho(\psi-1)}{4(C(\psi)-\rho)} + k - 1, \frac{(5-\psi)\rho}{4(C(\psi)-\rho)} + 5 - k\right)}{\mathcal{B}\left(\frac{(\psi-1)\rho}{4(C(\psi)-\rho)}, \frac{(5-\psi)\rho}{4(C(\psi)-\rho)}\right)} \quad (2)$$

GSD Examples



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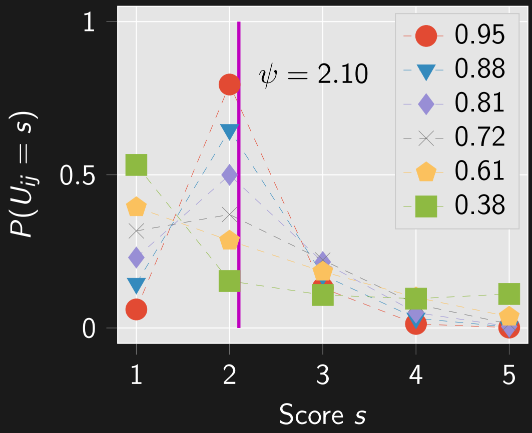

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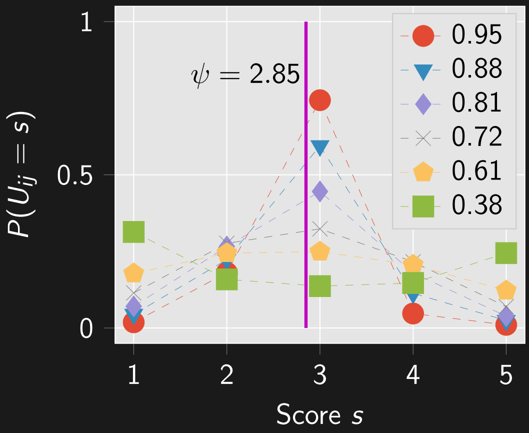

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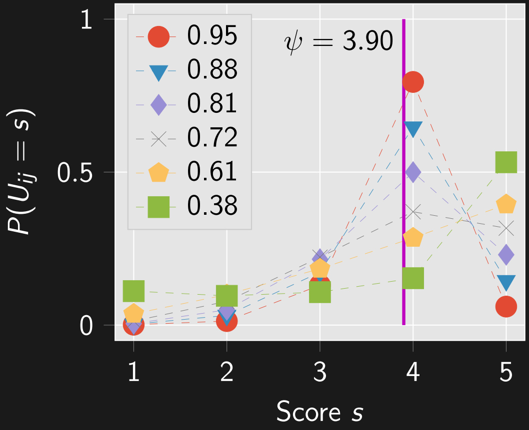

Parameters Estimation

Simulation:

- a number of subjects $N = \{6, 12, 24, 48\}$,
- ψ from 1.05 to 4.95 (with 23 different values),
- ρ from 0.01 to 0.99 (with 23 different values).

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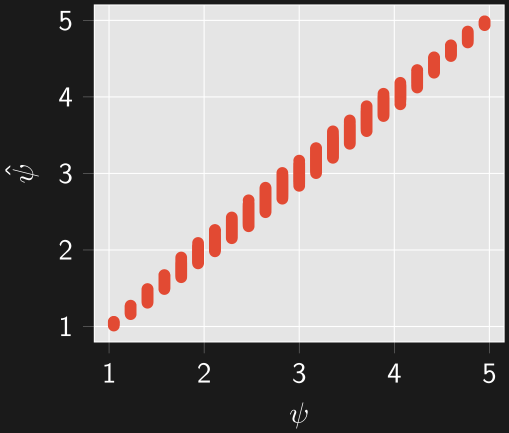

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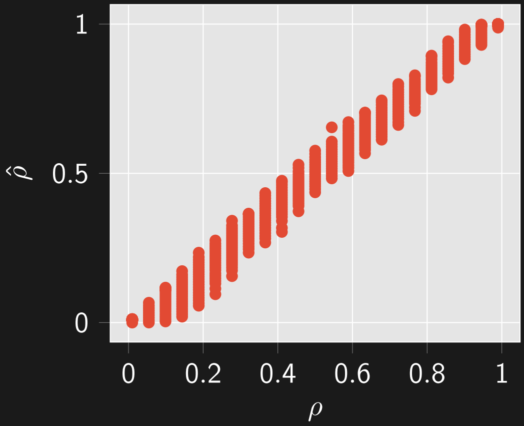
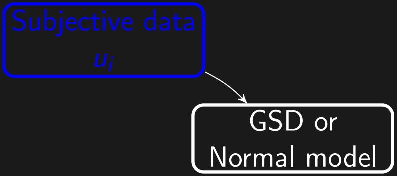


Testing Distribution

Subjective data
u_i

Testing Distribution



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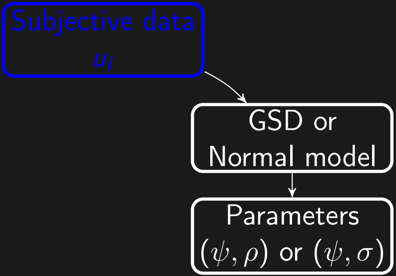

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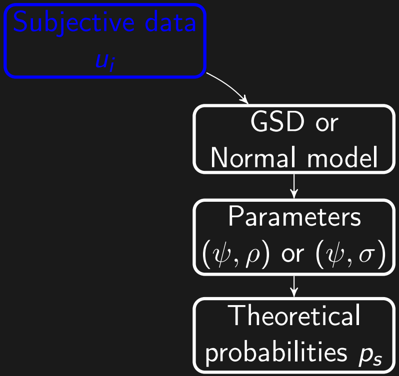

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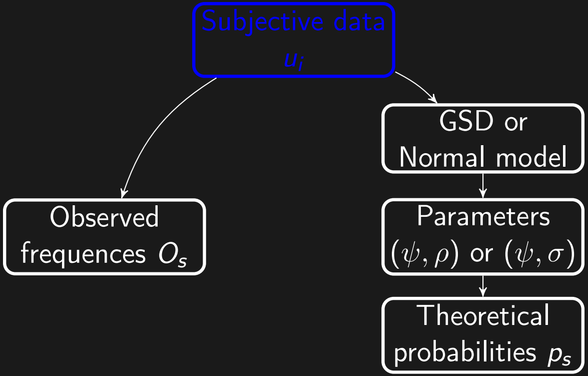

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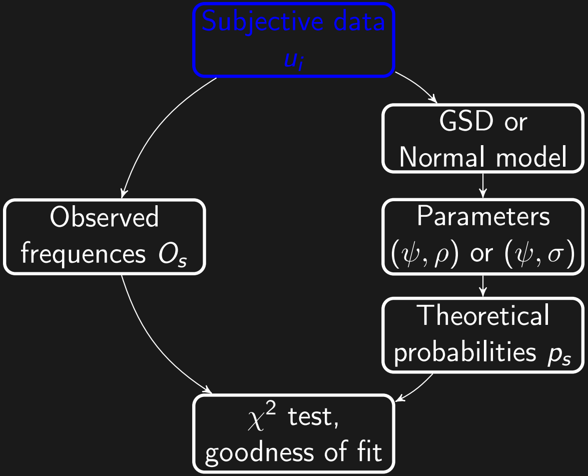

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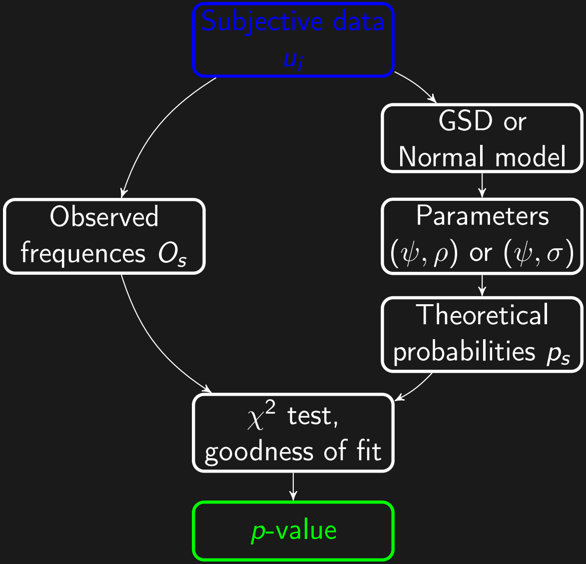
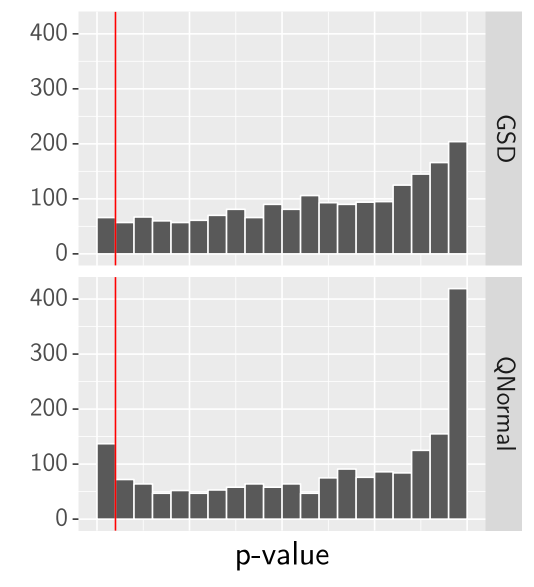
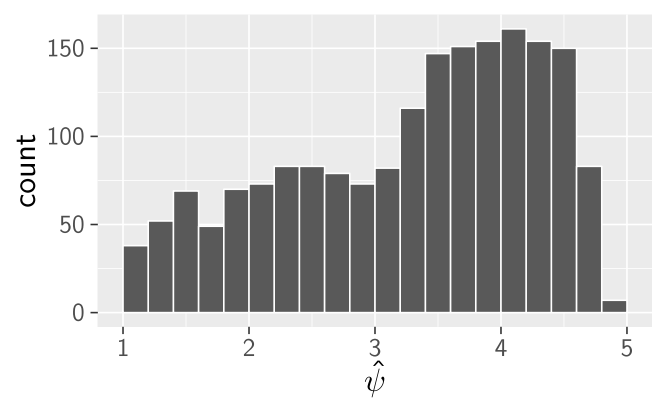


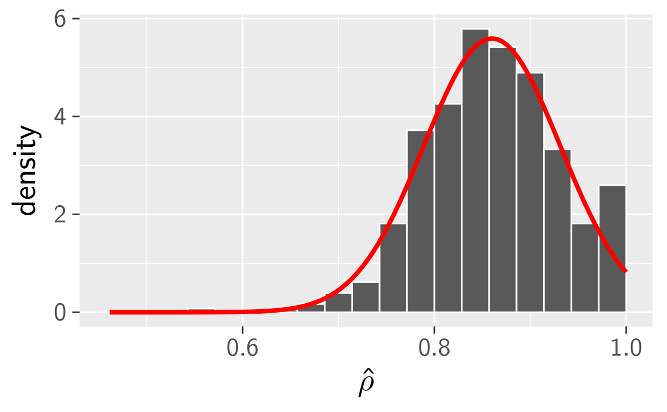
Results for 1874 Sequences



Prior Distribution ψ



Prior Distribution ρ



More Details

The full paper describing the GSD can be found here: https://arxiv.org/pdf/1909.04369.

Further Steps

- Use GSD for bootstrap
- Advance the GSD to model taking into account subject bias or error coming from subjects and work on better estimation method like Bayes
- Use GSD for different data like not pixel quality but whole movie (like imdb) or product quality
- Crate a correct test with GSD so we can test influencing factors like if gender influences the quality score
- Test which scale is correct by analyzing the answer spread

Questions